Loria - Inria, Nancy (France)
ENS Paris-Saclay (France)

Descriptive complexity on non-Polish spaces

Antonin Callard & Mathieu Hoyrup

STACS, March 2020

A motivating example

$$\text{Let } A = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\} \subseteq \mathbb{R}$$

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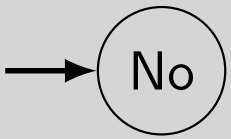

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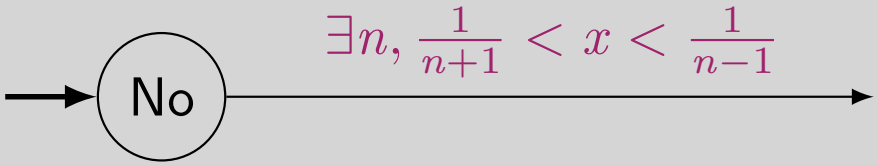

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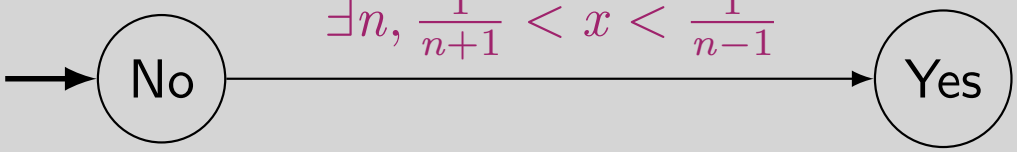

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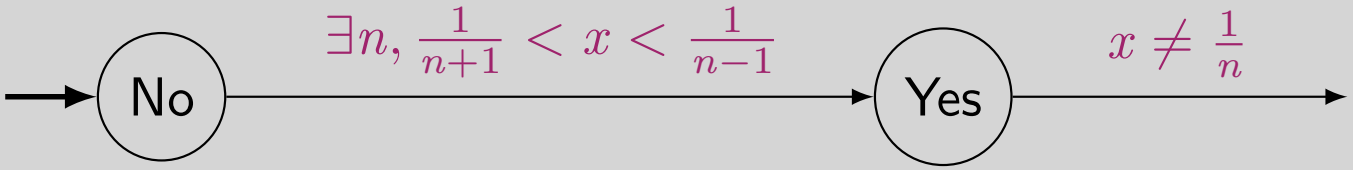

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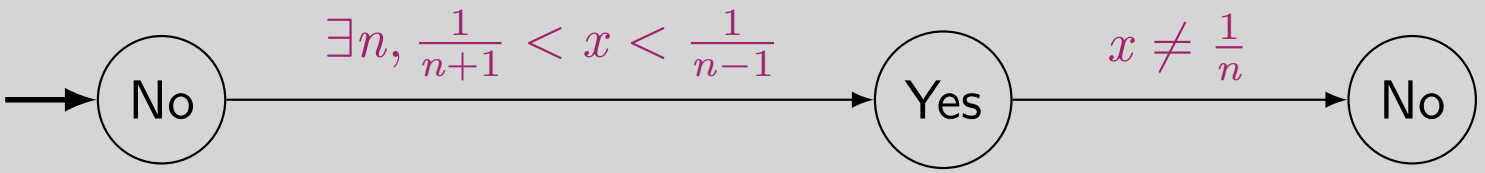

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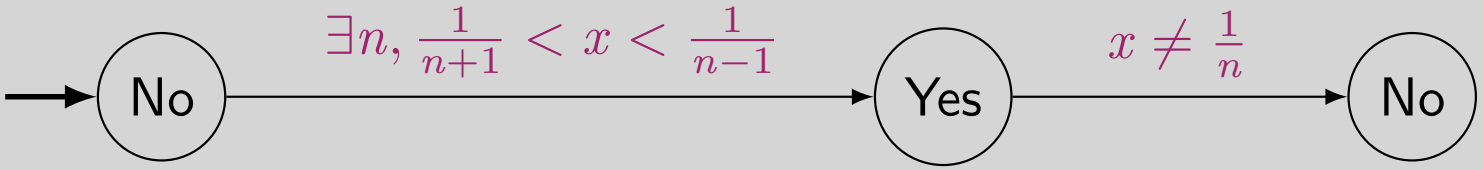

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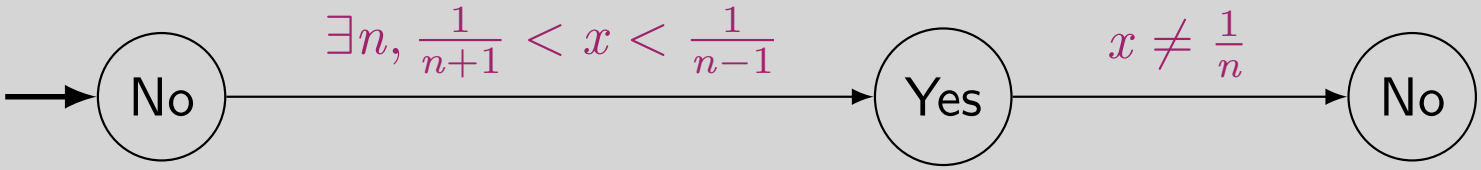

Topological description:

$$A = (0, +\infty) \setminus \bigcup_{n \in \mathbb{N}} \left(\frac{1}{n+1}, \frac{1}{n} \right)$$

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$$\iff$$
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Question:

Are those two approaches always equivalent?

Outline

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Formalization of the problem

Countably-based spaces

Non countably-based spaces

Conclusion

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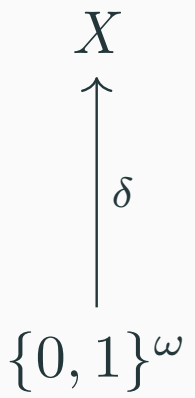

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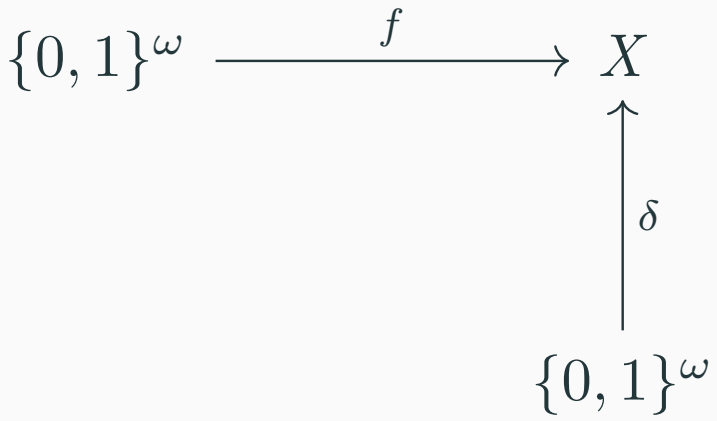

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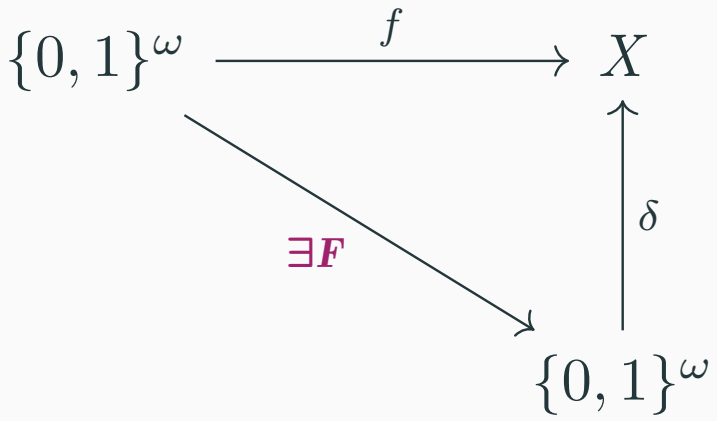

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Key idea: semi-decidable problems are similar to (effective) open sets!

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- $[\Pi_1^0]$: problems P verifying $P^c \in [\Sigma_1^0]$, ie. :

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$[\Delta_2^0]$ = **converging sequences**.

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$\Delta_2^0 = \Sigma_2^0 \cap \Pi_2^0$,
where $\Sigma_2^0 = \{\bigcup_{n\in\mathbb{N}} U_n \setminus U'_n\}$.

Reformulation of the problem

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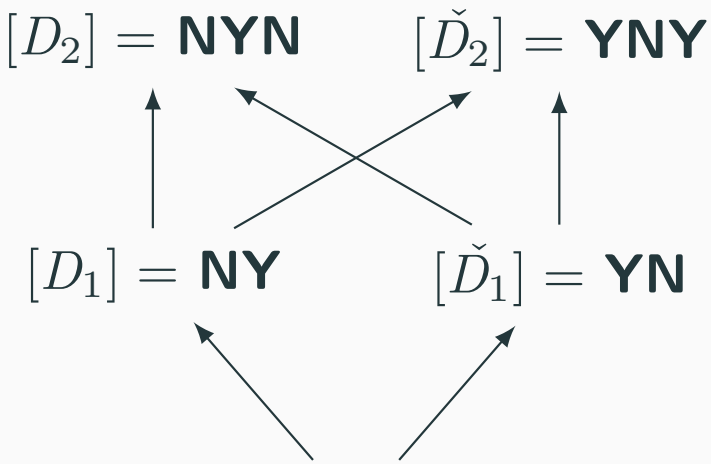

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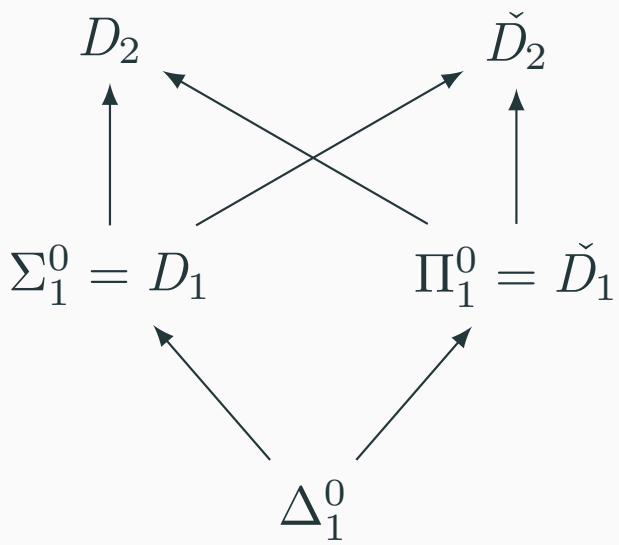

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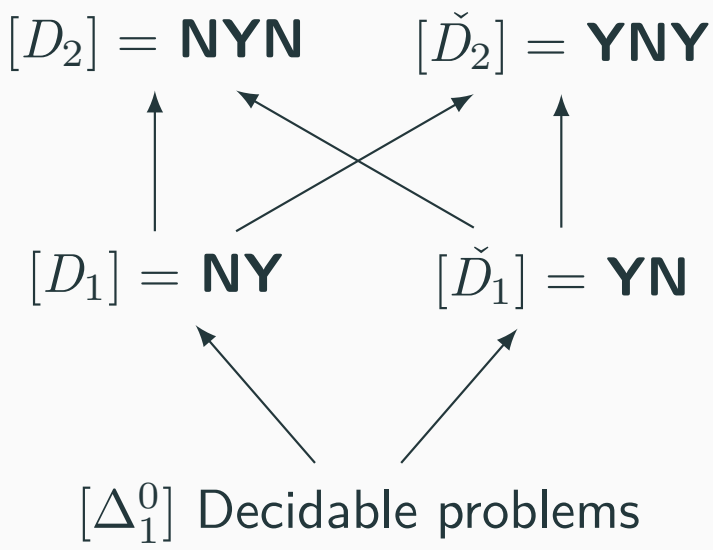

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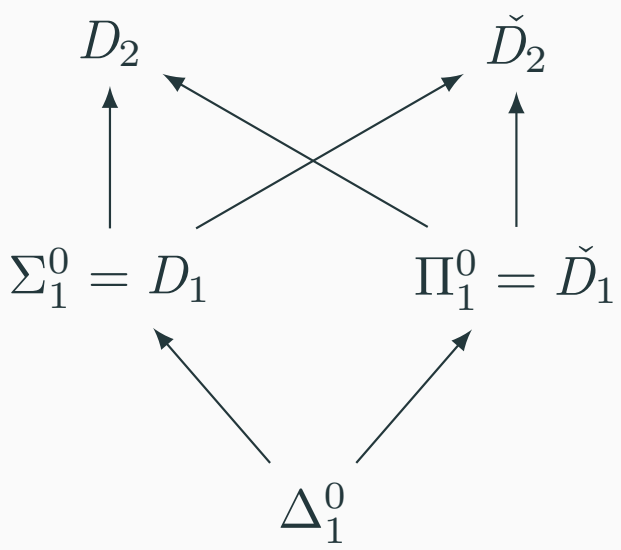

For $\Gamma \in \{D_\eta : \eta < \omega_1\}$, when does one have $\Gamma = [\Gamma]$?

Countably-based spaces

Countably-based spaces: matching of the two hierarchies

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Theorem 2.1: M. De Brecht, 2012

For (X, δ) a countably-based represented space,

$$S \in \Gamma \iff S \in [\Gamma]$$

Countably-based spaces: matching of the two hierarchies

Countably-based: a countable number of open sets describes the topology.

[New] - Theorem 2.2: Hierarchies coincide in an effective way

For (X, δ) a countably-based represented space,

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in an effective and uniform way.

Countably-based: about hardness

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[New] - Theorem 2.6: Hardness-criterion

$$S \notin D_\eta \iff S \text{ is } \check{D}_\eta\text{-hard.}$$

Countably-based: about hardness

Theorem 2.7: Hardness-criterion

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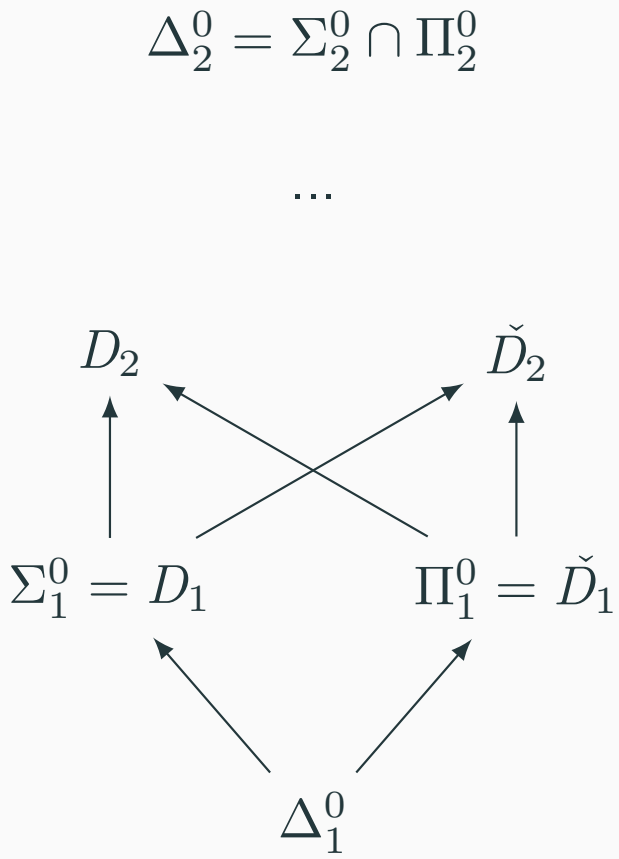

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- The coefficients $p_0, ..., p_n$ such that
$$P = p_0 + p_1 X + ... + p_n X^n.$$

Complexity of S_1

Property 3.1: Complexity of S_1 (Part 1)

$$S_1 = \left\{ P \in \mathbb{R}[X] : p_0 = 0 \text{ or } p_0 > \frac{1}{\deg(P)} \right\} \in [D_2]$$

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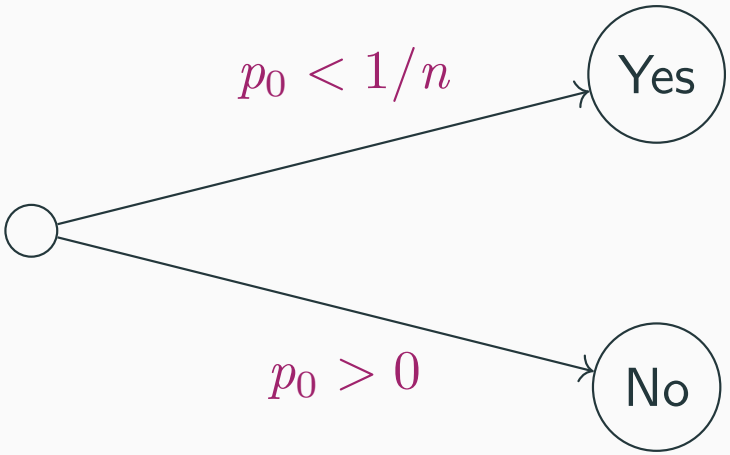

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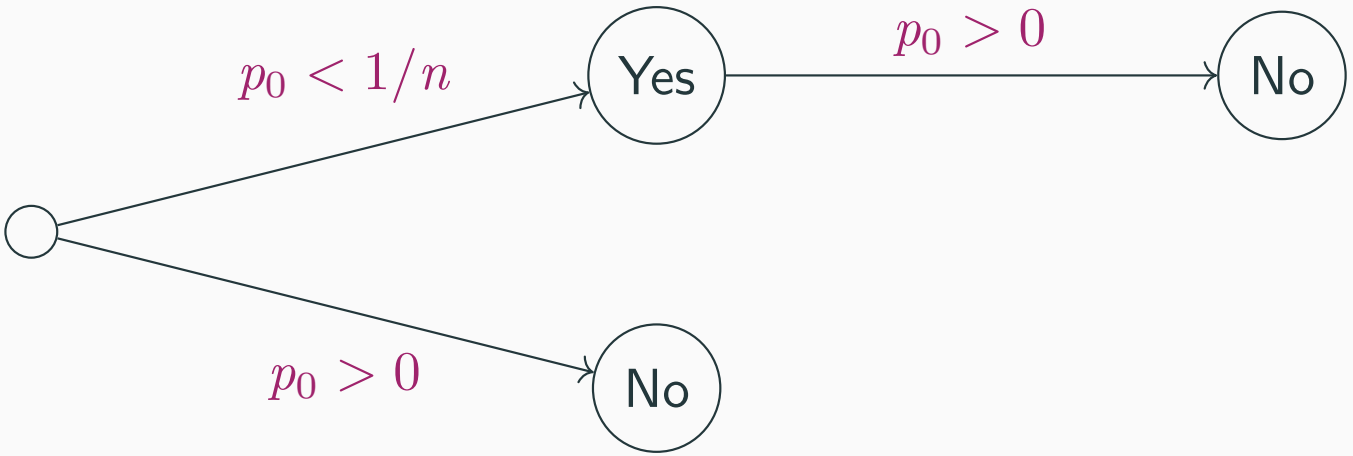

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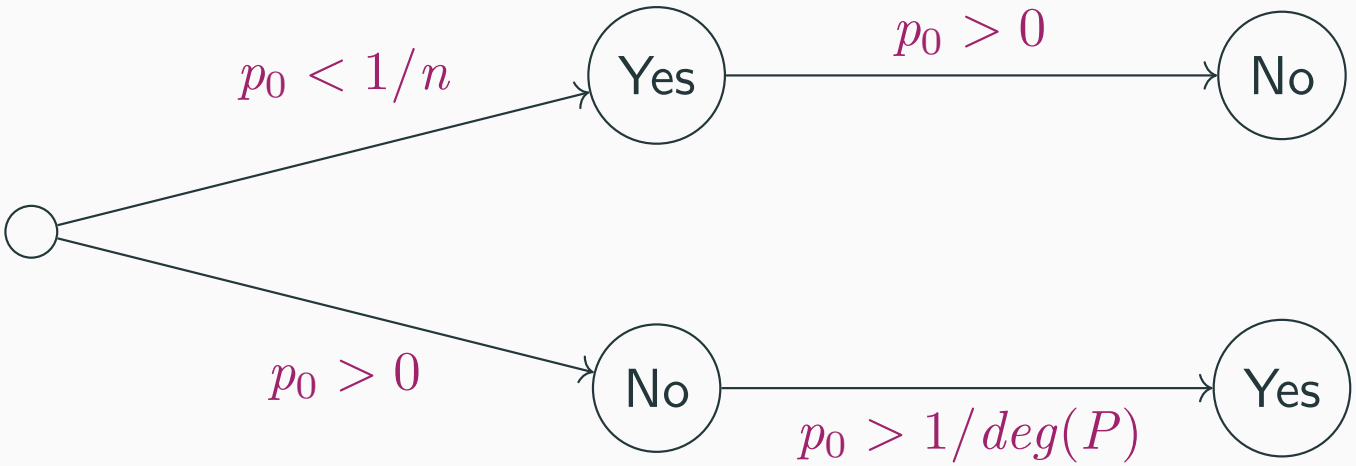

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$$\frac{1}{n} + \frac{X^{n+1}}{p} \ (\in S_1) \xrightarrow[p \mapsto +\infty]{} \frac{1}{n} \ (\notin S_1)$$

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(ie. S_1 is not a difference of two open sets)

Proof:

$$\frac{1}{n} + \frac{X^{n+1}}{p} \ (\in S_1) \xrightarrow[p \mapsto +\infty]{} \frac{1}{n} \ (\notin S_1) \xrightarrow[n \mapsto +\infty]{} 0 \ (\in S_1)$$

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Conclusion : $S_1 \in [D_2]$ but $S_1 \notin D_2$

Complexity of S_1

$[\Delta_2^0] =$ converging sequences

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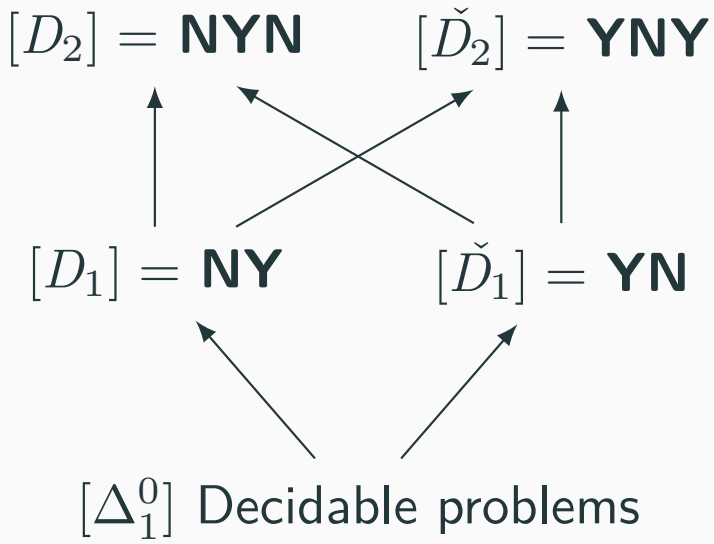

$\Delta_2^0 = \Sigma_2^0 \cap \Pi_2^0$

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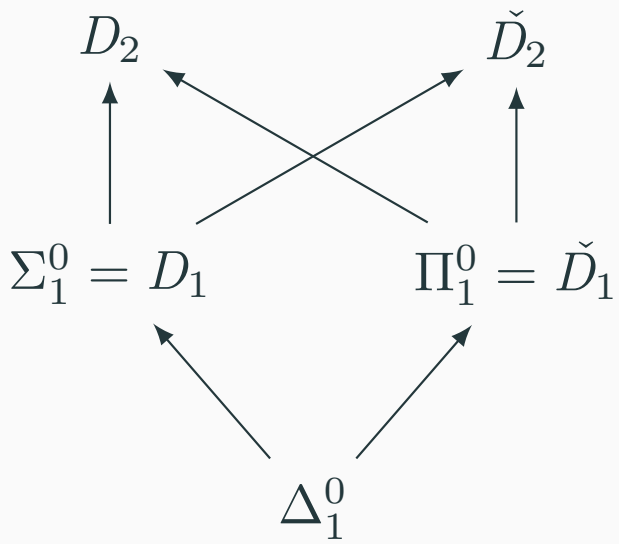

To go even further... Introduce S_2.

$$S_2 = \left\{ \frac{1}{k_1} + \frac{X^{k_1}}{k_2} + ... + \frac{X^{k_n}}{k_{n+1}} : \exists n, k_1 < ... < k_{n+1}, k_n \text{ even} \right\}$$

[New] - Theorem 3.3: Complexity of S_2

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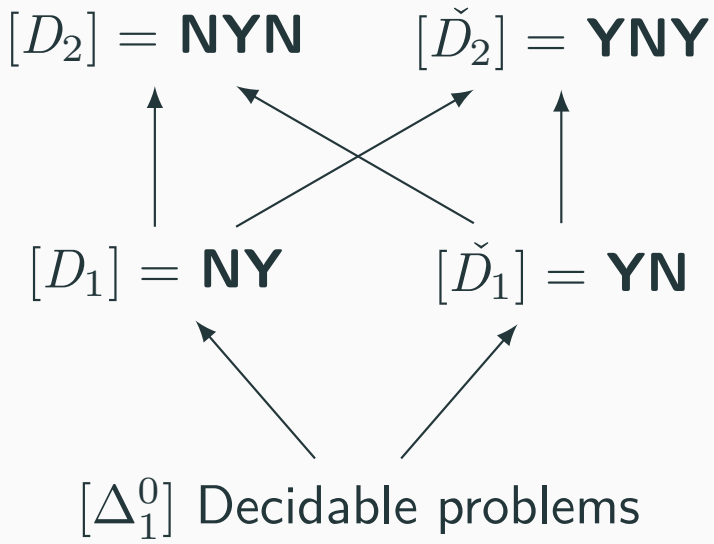

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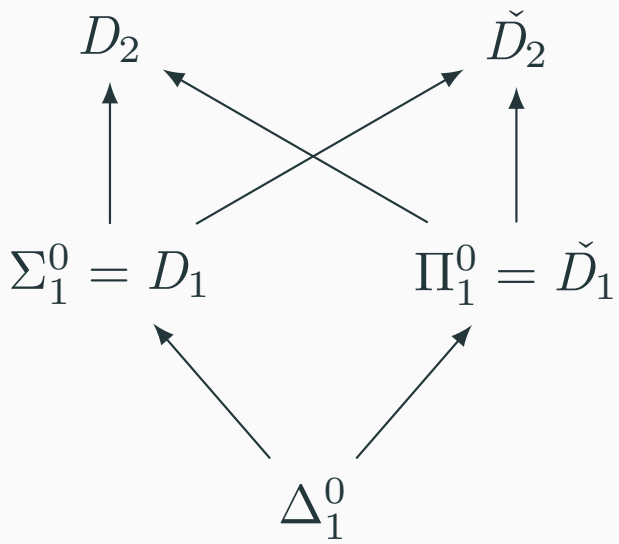

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Guess: The difference is related to the mismatch between **topological** and **sequential** aspects.

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Why do we think that?

[New] - Theorem 3.4: $[\Gamma]$ depends on sequentiality

For a coPolish space X, 1. and 2. are equivalent.

1. Closure and sequential closure coincide on X.
2. For every $n < \omega$, $A \in [D_n]$ iff $A \in D_n$.

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For example:

[New] Theorem 3.5: Countably-based spaces

$$A \notin D_\eta \iff A \text{ is } \check{D}_\eta\text{-hard.}$$

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DST works better with algorithmic complexity (and represented spaces) rather than with topology.

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Let us recap:

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Open questions: Why is the equivalence broken? On which spaces does it happen, at which complexity levels, and why?

Our guess: The difference is related to the mismatch between **topological** and **sequential** aspects.

Questions?